Thermal Radiation Effects on Boundary Layer Flow and Gas Concentration over a Linear Stretching Surface

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Abstract - The steady two-dimensional laminar boundary layer flow and heat transfer of a viscous incompressible electrically conducting fluid over a linear stretching surface in the presence of a uniform magnetic field with thermal radiation are investigated. The governing boundary layer equations are transformed to ordinary differential equations by compelling suitable similarity transformation and solved numerically by maple software using rk-45 method. The effects of various parameters such as magnetic parameter, radiation parameter, Prandtl number, thermal conductivity factor and Eckert number on velocity, temperature distributions and gas concentration are computed and represented graphically.

Keywords - Magneto hydrodynamics; Thermal radiation; stretching sheet; Uniform Magnetic Field; Gas concentration.

I. INTRODUCTION

For many years, engineers and scientists are concerned with the flow of fluids which have got many important applications in include soil erosion by natural winds and dust, entertainment in a cloud during nuclear explosion. In petroleum industry, purification of crude oil, electrostatic precipitation, polymer technology, transport processes shock waves. Also such flows occur in a wide range of areas of technical importance like fluidization, flow in rocket tubes, combustion, paint spraying and more recently blood flows in capillaries, fluidized beds, combustion, and gas cooling systems, centrifugal separation of matter from fluid. Saffman [1] studied the work by formulating the governing equations for the flow of dusty fluid. Some representative studies in this direction can be refereed to Refs [2]-[4]. Vajravelu and Neyfeh [5] analyzed the hydro magnetic flow of a dusty fluid over a stretching sheet with the effect of suction. Gireesha et al. [6] investigated the effect of non-uniform source/sink on flow and heat transfer of dusty fluid over a stretching sheet. Further Ramesh et al. [7] studied the magnetic field and non-uniform source/sink properties on stagnation point flow of a dusty fluid over a stretching sheet. Recent works of [8]-[10] have been done in area of dusty flow over a stretching sheet or plate under different effects and conditions. Crane [11] was the first to consider the boundary layer flow caused by a stretching sheet which moves with a velocity varying linearly with the distance from a fixed point. The heat transfer aspect of this problem was investigated by Carragher and Crane [12], under the conditions when the temperature difference between the surface and the ambient fluid is proportional to a power of the distance from a fixed point. R.L.V. Renuka Devi at al.[13] studied the Radiation an Mass Transfer Effects on MHD Boundary Layer Flow due to an Exponentially Stretching Sheet with Heat Source. Mamatha Upadhya, Mahesha and C.S.K. Raju [14] are investigated Cattaneo-Christov on heat and mass transfer of unsteady Eyring Powell dusty nanofluid over sheet with heat and mass flux conditions. And Mamatha S.U. Mahesh at al [15] premeditated Induced Magnetic Field on Williamson Fluid Past a Stretching Surface with Nonlinear Thermal Radiation and Non-Uniform Heat Source or Sink. Prasannakumara at al.[16] deliberated Melting Phenomenon in MHD Stagnation Point Flow of Dusty Fluid over a Stretching Sheet in the Presence of Thermal Radiation and Non-Uniform Heat Source/Sink. M.S. Abela and Mahesha N [17] investigated the heat transfer in MHD viscoelastic fluid flow over a stretching sheet with variable thermal conductivity, non-uniform heat source and radiation, author Elabashbeshy E.M.A [18] have discussed Heat and mass transfer along a vertical plate with variable temperature and concentration in the presence of magnetic field. Sahoo, P.K., Datta, N. and Biswal,.S [19] studied the MHD unsteady free convection flow past an infinite vertical plate with constant suction and heat sink, Poornima, T. and Bhaskar Reddy, N [20] discussed the concept of Radiation effects on MHD free convective boundary layer flow of nanofluids over a nonlinear stretching sheet. Rashidi et al [21] have investigated the combined effect of magnetic field and thermal radiation over a vertical stretching sheet for two dimensional water based nanofluid flow. Shanker, Kishan [22] are investigated the effect of mass transfer on the MHD flow past an impulsively started infinite vertical plate. Siddheshwar and Mahabaleswar [23] studied the effects of radiation and source on MHD flow of a viscoelastic liquid and heat transfer over a stretching sheet. Kameswaran et al [24] studied the heat and mass transfer effects on MHD Newtonian liquid flow over an exponentially stretching in presence of radiation. Jat and Gopi Chand [25] proposed the effects of dissipation and radiation on MHD flow and heat transfer over an exponentially stretching sheet. Chaim [26] investigated the heat transfer in fluid flow Prandtl number with variable thermal conductivity, induced due to stretching sheet and compared the numerical results with perturbation solution. Kandasamya et at [27] determined the scaling group transformation for MHD boundary layer flow of a nanofluid past a vertical stretching surface in the presence of suction/ injection. Paresh Vyas and Nupur Srivastava [28] present a numerical study for the steady two-dimensional radiative MHD boundary layer flow of an incompressible, viscous, electrically conducting fluid caused by a non-isothermal linearly stretching sheet placed at the bottom of fluid saturated porous medium. By referring these articles the objective of the present paper is to investigate the effect of thermal radiation and heat transfer on the steady MHD boundary layer flow due to a linear stretching sheet in the presence of uniform heat source. The governing boundary layer equations have been transformed to a two point boundary value problem in similarity variables and the resultant problem is solved numerically by a Boundary Value Problem Method. The variation of velocity and radiation over non-dimensional parameters are calculated and depicted.

II. PROBLEM FORMULATION

Consider the x- axis is taken along the stretching surface in the direction of motion and y-axis is taken perpendicular to it. The stretching surface has a uniform temperature $T_w(x) = T_\infty + T_0\left(\frac{x}{L}\right)$ and a linear velocity $U_w(x) = b\left(\frac{x}{L}\right)$ while temperature of flow external to the boundary layer is T_∞. The system of governing boundary layer equations are given by:

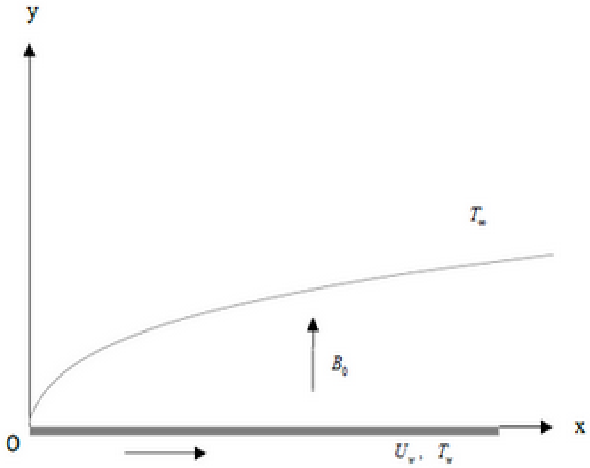

Fig. 1 Stretching surface representation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 u}{\rho} \qquad (2)$$

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\,\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 - \frac{\partial q_r}{\partial y} \qquad (3)$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} \qquad (4)$$

The boundary conditions for velocity, temperature and concentration fields are:

$$u = U_w(x) \quad v = 0 \quad T = T_w(x) \quad C = C_w(x) \;\; at \;\; y = 0 \qquad (5)$$
$$u \to 0 \quad T \to T_\infty \;\; C \to C_\infty \;\; at \;\; y \to \infty$$

Where u and v are the velocity components along the x and y axes, respectively, T_0 is the reference temperature, L is the reference length, $\upsilon = \frac{\mu}{\rho}$ is the coefficient of kinematic viscosity, μ is the coefficient of viscosity, ρ is the fluid density, σ is the electrical conductivity, C_p is the specific heat at constant pressure, T is the temperature, C is the fluid concentration, T_w is the temperature of sheet, C_w is the concentration of the sheet, k is the thermal conductivity and q_r is the radiative heat flux and D is the coefficient of mass diffusivity, b is the stretching constant with $b > 0$.

By using Rosseland approximation, the radiative heat flux q_r is expressed as

$$q_r = \frac{-4\rho^*}{3K^*}\frac{\partial T^4}{\partial y} \qquad (6)$$

Where ρ∗ is the Stefan-Boltzmann constant and K^* is the mean absorption coefficient. The above radiative heat flux q_r is effective at a point away from boundary layer surface in an intensive absorption flow. Considering that the temperature difference within the flow is sufficiently small, the T^4 may be expressed as a linear function of temperature T. Expanding T^4 by Taylor's series about temperature T_∞ and which after neglecting higher order terms takes the forms,

$$T^4 \cong 4T_\infty^3\, T - 3\, T_\infty^4 \tag{7}$$

Using equation (6) and (7) in equation (3) is reduced to:

$$\rho C_p \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \left(K + \frac{16\rho^* T_\infty^3}{3k^*}\right)\frac{\partial^2 T}{\partial y^2} \tag{8}$$

Similarity analysis: The continuity equation (1) satisfied by the Cauchy Riemann equations,

$$u = \frac{\partial \psi}{\partial y}\ ,\quad v = -\frac{\partial \psi}{\partial x} \tag{9}$$

For the solution of momentum and energy equations, introducing the following dimensionless variables:

$$\psi(x,y) = \sqrt{2\upsilon L U_w}\, f(\eta) \tag{10}$$

$$\eta = \sqrt{\frac{U_w}{2\upsilon L}}\, y \tag{11}$$

$$T = T_\infty + T_0\left(\frac{x}{L}\right)\theta(\eta) \tag{12}$$

$$C = C_\infty + C_0\left(\frac{x}{L}\right)\phi(\eta) \tag{13}$$

Where $f(\eta)$ is the dimensionless stream function, θ is the dimensionless temperature, ϕ is the dimensionless concentration, η - the similarity variable.
In view of Equations (9) to (13) in equation (2), (4) and (8) reduced to:

$$f''' - \frac{2b}{U_w}f'^2 + \frac{b}{U_w}ff'' - Mf' = 0 \tag{14}$$

$$\left(1 + \frac{4}{3}R\right)\theta'' + Prf\theta' - 2Pr\theta f' + Ecf'' = 0 \tag{15}$$

$$\phi'' - 2R\phi f' + R\phi' f = 0 \tag{16}$$

Where $M = \frac{2\sigma B_0^2 L}{\rho U_w}$ is the Magnetic Parameter, $Pr = \frac{\rho v C_p b}{U_w k}$ is the Prandtl number, and $R = \frac{Scb}{U_w C_0}$ is the Radiation parameter, Sc– the Schmidt number and $Ec = \frac{\mu U_w b}{kT_0}$ is the Eckert number.
The transformed boundary conditions are:

$$f = 0,\quad f' = 1,\ \theta = 1,\ \phi = 1 \quad at\ \ \eta = 0$$
$$f' = 0,\quad \theta = 0,\quad \phi = 0 \quad at\ \ \eta = \infty \tag{17}$$

III. RESULTS AND DISCUSSION

Figure 2 shows the variation of the velocity profile $f'(\eta)$ against η.As magnetic parameter M increases velocity profile decreases due to the effect of Lorentz force produced by transverse magnetic field causes deceleration of fluid velocity.
Figures 3 to 6 shows the temperature distribution $\theta(\eta)$ against η for various values of the magnetic parameter M, the thermal conductivity k, the Eckert number Ec and the Prandtl number Pr. We observed from these pictures that the temperature distribution $\theta(\eta)$ decreases with increasing value of any parameter, such as the magnetic parameter M, the thermal conductivity k, the Eckert number Ec and the radiation parameter R. However, it decreases with increasing value of the Prandtl number Pr. An increasing the Prandtl number Pr, causes decrease in thermal boundary layer of fluid flow.
Figure-7 shows the concentration distribution $\phi(\eta)$ against η for various values of the radiation parameter R.As temperature increases the concentration of gas decreasing. Table I shows the variation of the velocity profile and temperature are numerically documented for different values of non- dimensional quantities, Eckert number Ec and Prandtl number Pr, magnetic parameter M, radiation parameter R and the thermal conductivity k are recorded.

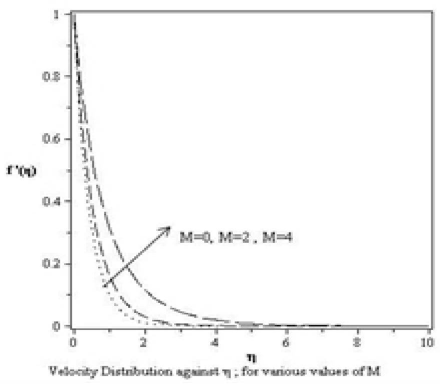

Figure 2: Velocity for different values of M

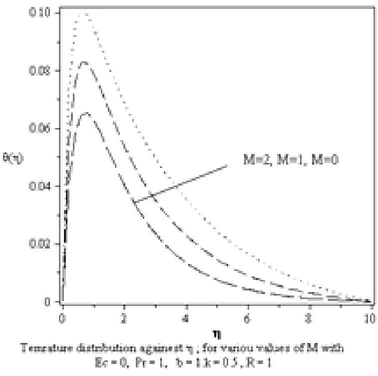

Figure 3: Temperature for different values of M

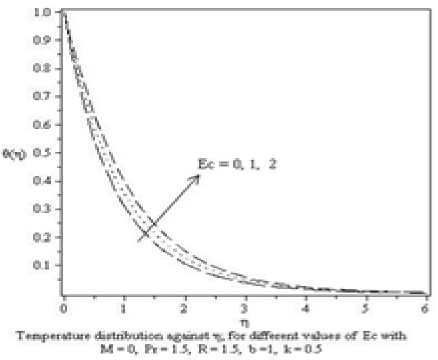

Figure 4: Temperature for different values of Ec

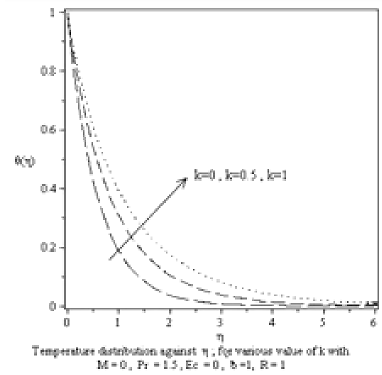

Figure 5: Temperature for different values of k

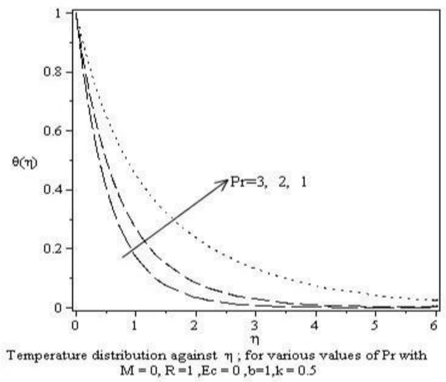

Figure 6: Temperature for different values of Pr

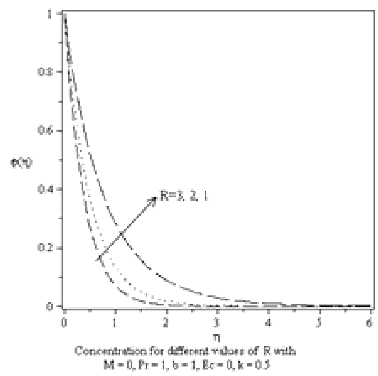

Figure 7: Concentration for different values of R

IV. CONCLUSION

In this present work we study the steady of boundary layer and heat transfer past of a stretching sheet with temperature dependent velocity is considered in the presence of thermal radiation heat source/sink and mass transfer. The governing equations approximated to a system of non-linear ordinary differential equation by similarity transformations using the maple software. The numerical calculations are carried out for different values of dimensionless parameter of the problem. The present solutions are authorized by comparing with the exiting solutions. Our result shows a good agreement with the existing work in the literature. The results are summarized as follows. (1) The velocity profile degenerates with the effect of Lorentz force, (2) Magnetic field uplifts the temperature and concentration but reduces the velocity, (3) The radiation enhances the velocity and temperature., and (4) he heat source/sink improves the velocity and temperature.

TABLE I: NUMERICAL VARIATION OF $f'(0)$ AND $\theta(0)$ FOR DIFFERENT VALUES FOR M, PR, EC AND K

M	K	Ec	R	Pr	$f'(0)$	$\theta(0)$
0	0.5	0	1	1	0.01634	0.02313
1	0.5	0	1	1	0.00149	0.07004
2	0.5	0	1	1	0.00027	0.11040
0	**0**	1	1	1.5	0.00714	0.00085
0	**0.5**	1	1	1.5	0.00714	0.01103
0	**1**	1	1	1.5	0.00714	0.03409

0	0.5	**0**	1	1	0.00714	0.01102
0	0.5	**1**	1	1	0.00714	0.01344
0	0.5	**2**	1	1	0.00714	0.01585
0	0.5	0	**1**	1	0.01634	0.02313
0	0.5	0	**2**	1	0.01634	0.02313
0	0.5	0	**3**	1	0.01634	0.02313
0	0.5	1	1	**1**	0.07211	0 .13743
0	0.5	1	1	**2**	0.01111	0.025251
0	0.5	1	1	**3**	0.00101	0.003703

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